Rare radiative B decays to orbitally excited K mesons

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The exclusive rare radiative B meson decays to orbitally excited axial-vector mesons $K_1^*(1270)$, $K_1(1400)$, and to the tensor meson $K_2^*(1430)$ are investigated in the framework of the relativistic quark model based on the quasipotential approach in quantum field theory. These decays are considered without employing the heavy quark expansion for the s quark. Instead the s quark is treated to be light and the expansion in inverse powers of the large recoil momentum of the final K^{**} meson is used to simplify calculations. It is found that the ratio of the branching fractions of rare radiative B decays to axial vector $K_1^*(1270)$ and $K_1(1400)$ mesons is significantly influenced by relativistic effects. The obtained results for B decays to the tensor meson $K_2^*(1430)$ agree with recent experimental data from CLEO and Belle.

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I. INTRODUCTION

Rare radiative decays of B mesons represent an important test of the standard model of electroweak interactions. These transitions are induced by flavor changing neutral currents and thus they are sensitive probes of new physics beyond the standard model. Such decays are governed by one-loop (penguin) diagrams with the main contribution from a virtual top quark and a W boson. Therefore, they provide valuable information about the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V_{ts} and V_{tb}. The statistics of rare radiative B decays has considerably increased since the first observation of the $B\to K^*\gamma$ decay in 1993 by CLEO [1]. This allowed a significantly more precise determination of exclusive and inclusive branching fractions [2]. The first observation of the rare B decays to the orbitally excited strange mesons has been reported by CLEO [2]. The branching fraction for the decay to the tensor $K_2^*(1430)$ meson has been measured $\mathrm{Br}(B\to K_2^*(1430)\gamma)=(1.66^{+0.59}_{-0.53}\pm 0.13)\times 10^{-5}$, as well as the ratio of exclusive branching fractions $r\equiv \mathrm{Br}(B\to K_2^*(1430)\gamma)/\mathrm{Br}(B\to K^*(892)\gamma)=0.39^{+0.15}_{-0.13}$. Recently Belle [3] published the measurement of the corresponding branching fraction $\mathrm{Br}(B\to K_2^*(1430)\gamma)=(1.89\pm 0.56\pm 0.18)\times 10^{-5}$, which is in good agreement with the CLEO result. The data for the other decay channels will be available soon. This significant experimental progress provides a challenge to the theory. Many theoretical approaches have been employed to predict the exclusive $B\to K^*(892)\gamma$ decay rate (for a review see Ref. [4], and references therein). Considerably less attention has been paid to rare radiative B decays to excited strange mesons [5–8]. Most of these theoretical approaches [6,8] rely on the heavy quark limit both for the initial b and final s quarks and the nonrelativistic quark model. However, the two predictions [6,8] for the ratio r differ by an order of magnitude, due to a different treatment of the long distance effects and, as a result, a different determination of the corresponding Isgur-Wise functions. Only the prediction of Ref. [8] is consistent with the available data. Nevertheless, it is necessary to point out that the s quark in the final K^* meson is not heavy enough, compared to the $\bar{\Lambda}$ parameter, which determines the scale of $1/m_Q$ corrections in heavy quark effective theory [9]. Thus the $1/m_s$ expansion is not appropriate. Notwithstanding, the ideas of heavy quark expansion can be applied to the exclusive $B\to K^*(K^{**})\gamma$ decays. From the kinematical analysis it follows that the final $K^*(K^{**})$ meson bears a large relativistic recoil momentum $|\mathbf{\Delta}|$ of order of $m_b/2$ and an energy of the same order. So it is possible to expand the matrix element of the effective Hamiltonian both in inverse powers of the b quark mass for the initial state and in inverse powers of the recoil momentum $|\mathbf{\Delta}|$ for the final state [10,11]. Such an expansion has been realized by us for the $B\to K^*(892)\gamma$ decay in the framework of the relativistic quark model [10]. In Ref. [11] it was shown that in the leading order of this expansion a specific symmetry emerges which imposes several relations between the form factors of semileptonic and rare radiative B decays. The interactions with collinear gluons preserve these relations, establishing them in the large energy limit of QCD [12]. It is important to note that rare radiative decays of B mesons require a completely relativistic treatment, because the recoil momentum of the final meson is large compared to the s quark mass. The calculated branching fraction for this decay [10] was found in good agreement with experimental data. In Ref. [13] we considered the exclusive rare B decay to the orbitally excited tensor meson $K_2^*(1430)$. Here we extend this analysis to the axial-vector mesons $K_1^*(1270)$ and $K_1(1400)$.

Our relativistic quark model is based on the quasipotential approach in quantum field theory with a specific choice of the quark-antiquark interaction potential. It provides a consistent scheme for the calculation of all relativistic corrections at a given v^2/c^2 order and allows for the heavy quark $1/m_Q$ expansion. In preceding papers we applied this model to the calculation of the mass spectra of orbitally and radially excited states of heavy-light mesons [14], as well as to the description of weak decays of B mesons to ground state heavy and light mesons [15,16]. The heavy quark expansion for the heavy-to-heavy semileptonic transitions [17,18] was found to be in agreement with model-independent predictions of the heavy quark effective theory (HQET).

The paper is organized as follows. In Sec. II we define the form factors, which govern the exclusive rare radiative B decays to orbitally excited K mesons. The relativistic quark model is described in Sec. III, and in Sec. IV it is applied for the calculation of the rare radiative decay form factors. Our numerical results for the form factors and decay rates as well as comparison of these results with other theoretical predictions and experimental data are presented in Sec. V. There we also discuss the relations between the form factors of rare radiative and semileptonic B decays in the formal heavy quark limit. Section VI contains our conclusions.

II. RARE RADIATIVE B DECAYS

In the standard model, B decays are described by the effective Hamiltonian, after integrating out the top quark and W boson and using the Wilson expansion [19]. For the case of $b\to s$ transition

$$H_{\rm eff}(b\to s)=-\frac{4G_F}{\sqrt{2}}V_{ts}^*V_{tb}\sum_{j=1}^{8}C_j(\mu)O_j(\mu), \qquad (1)$$

where V_{ij} are the corresponding CKM matrix elements and $\{O_j\}$ are a complete set of renormalized dimension six operators involving light fields which govern $b\to s$ transitions. They include six four-quark operators O_j ($j=1,\ldots,6$), which determine the nonleptonic B decay rates, the electromagnetic dipole operator

$$O_7=\frac{e}{16\pi^2}\bar{s}\sigma^{\mu\nu}(m_bP_R+m_sP_L)bF_{\mu\nu}, \quad P_{R,L}=(1\pm\gamma_5)/2, \qquad (2)$$

and the chromomagnetic dipole operator O_8. O_7 and O_8 are responsible for the rare B decays $b\to s\gamma$ and $b\to sg$, respectively [19]. The Wilson coefficients $C_j(\mu)$ are evaluated perturbatively at the W scale and then they are evolved down to the renormalization scale $\mu\sim m_b$ by the renormalization group equations. There are also power-suppressed terms $\sim 1/m_c^2$ [20,21].

The dominant contribution to $B\to K^{**}\gamma$ decay rates come from the electromagnetic dipole operator O_7. The matrix elements of this operator between the initial B meson state and the final state of the orbitally excited K^{**} meson have the following covariant decomposition:

$$\langle K_1(p',\epsilon)|\bar{s}ik_\nu\sigma_{\mu\nu}b|B(p)\rangle$$
$$=t_+(k^2)((\epsilon^*\cdot k)(p+p')_\mu-\epsilon^*_\mu(p^2-p'^2))$$
$$+t_-(k^2)((\epsilon^*\cdot k)k_\mu-\epsilon^*_\mu k^2)+t_0(k^2)(\epsilon^*\cdot k)$$
$$\times((p^2-p'^2)k_\mu-(p+p')_\mu k^2)/(M_BM_{K_1}),$$

$$\langle K_1(p',\epsilon)|\bar{s}ik_\nu\sigma_{\mu\nu}\gamma_5b|B(p)\rangle$$
$$=it_+(k^2)\epsilon_{\mu\nu\lambda\sigma}\epsilon^{*\nu}k^\lambda(p+p')^\sigma, \qquad (3)$$

$$\langle K_2^*(p',\epsilon)|\bar{s}ik_\nu\sigma_{\mu\nu}b|B(p)\rangle$$
$$=ig_+(k^2)\epsilon_{\mu\nu\lambda\sigma}\epsilon^{*\nu\beta}\frac{p_\beta}{M_B}k^\lambda(p+p')^\sigma,$$

$$\langle K_2^*(p',\epsilon)|\bar{s}ik_\nu\sigma_{\mu\nu}\gamma_5b|B(p)\rangle$$
$$=g_+(k^2)\left(\epsilon^*_{\beta\gamma}\frac{p^\beta p^\gamma}{M_B}(p+p')_\mu-\epsilon^*_{\mu\beta}\frac{p^\beta}{M_B}(p^2-p'^2)\right)$$
$$+g_-(k^2)\left(\epsilon^*_{\beta\gamma}\frac{p^\beta p^\gamma}{M_B}k_\mu-\epsilon^*_{\mu\beta}\frac{p^\beta}{M_B}k^2\right)$$
$$+g_0(k^2)((p^2-p'^2)k_\mu-(p+p')_\mu k^2)\epsilon^*_{\beta\gamma}\frac{p^\beta p^\gamma}{M_B^2M_{K_2^*}}. \qquad (4)$$

The matrix element of $B\to K_1^*\gamma$ decay has the same decomposition as Eq. (3), with form factors $s_+(k^2)$, $s_-(k^2)$, $s_0(k^2)$ replacing form factors $t_+(k^2)$, $t_-(k^2)$, $t_0(k^2)$. Here ϵ_μ ($\epsilon_{\mu\nu}$) is a polarization vector (tensor) of the final axial-vector (tensor) meson and $k=p-p'$ is the four momentum of the emitted photon. The exclusive decay rates for the emission of a real photon ($k^2=0$) are determined by form factors $t_+(0)$, $s_+(0)$, and $g_+(0)$. They are given by

$$\Gamma(B\to K_1\gamma)=\frac{\alpha}{32\pi^4}G_F^2m_b^5|V_{tb}V_{ts}|^2|C_7(m_b)|^2t_+^2(0)$$
$$\times\left(1-\frac{M_{K_1}^2}{M_B^2}\right)^3\left(1+\frac{M_{K_1}^2}{M_B^2}\right), \qquad (5)$$

$$\Gamma(B\to K_1^*\gamma)=\frac{\alpha}{32\pi^4}G_F^2m_b^5|V_{tb}V_{ts}|^2|C_7(m_b)|^2s_+^2(0)$$
$$\times\left(1-\frac{M_{K_1^*}^2}{M_B^2}\right)^3\left(1+\frac{M_{K_1^*}^2}{M_B^2}\right), \qquad (6)$$

$$\Gamma(B\to K_2^*\gamma)$$
$$=\frac{\alpha}{256\pi^4}G_F^2m_b^5|V_{tb}V_{ts}|^2|C_7(m_b)|^2g_+^2(0)\frac{M_B^2}{M_{K_2^*}^2}$$
$$\times\left(1-\frac{M_{K_2^*}^2}{M_B^2}\right)^5\left(1+\frac{M_{K_2^*}^2}{M_B^2}\right), \qquad (7)$$

where $C_7(m_b)$ is the Wilson coefficient in front of the operator O_7. It is convenient to consider the ratio of exclusive to inclusive branching fractions, for which we have

$$R_{K_1}\equiv\frac{\mathrm{Br}(B\to K_1(1400)\gamma)}{\mathrm{Br}(B\to X_s\gamma)}=t_+^2(0)\frac{(1-M_{K_1}^2/M_B^2)^3(1+M_{K_1}^2/M_B^2)}{(1-m_s^2/m_b^2)^3(1+m_s^2/m_b^2)},\tag{8}$$

$$R_{K_1^*}\equiv\frac{\mathrm{Br}(B\to K_1^*(1270)\gamma)}{\mathrm{Br}(B\to X_s\gamma)}=s_+^2(0)\frac{(1-M_{K_1^*}^2/M_B^2)^3(1+M_{K_1^*}^2/M_B^2)}{(1-m_s^2/m_b^2)^3(1+m_s^2/m_b^2)},\tag{9}$$

$$R_{K_2^*}\equiv\frac{\mathrm{Br}(B\to K_2^*(1430)\gamma)}{\mathrm{Br}(B\to X_s\gamma)}=\frac{1}{8}g_+^2(0)\frac{M_B^2}{M_{K_2^*}^2}\frac{(1-M_{K_2^*}^2/M_B^2)^5(1+M_{K_2^*}^2/M_B^2)}{(1-m_s^2/m_b^2)^3(1+m_s^2/m_b^2)}.\tag{10}$$

The recent experimental values for the inclusive decay branching fraction

$$\mathrm{Br}(B\to X_s\gamma)=\begin{cases}(3.15\pm0.35\pm0.32\pm0.26)\times10^{-4} & \text{(CLEO [22])},\\ (3.39\pm0.53\pm0.42^{+0.51}_{-0.55})\times10^{-4} & \text{(Belle [3])}\end{cases}$$

are in a good agreement with theoretical calculations (see, e.g., Ref. [4]).

III. RELATIVISTIC QUARK MODEL

Now we use the relativistic quark model for the calculation of the form factors $t_+(0)$, $s_+(0)$, and $g_+(0)$. In our model a meson is described by the wave function of the bound quark-antiquark state, which satisfies the quasipotential equation [23] of the Schrödinger type [24] in the center-of-mass frame:

$$\left(\frac{b^2(M)}{2\mu_R}-\frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p})=\int\frac{d^3q}{(2\pi)^3}V(\mathbf{p},\mathbf{q};M)\Psi_M(\mathbf{q}),\tag{11}$$

where the relativistic reduced mass is

$$\mu_R=\frac{M^4-(m_q^2-m_Q^2)^2}{4M^3},\tag{12}$$

and $b^2(M)$ denotes the on-mass-shell relative momentum squared

$$b^2(M)=\frac{[M^2-(m_q+m_Q)^2][M^2-(m_q-m_Q)^2]}{4M^2}.\tag{13}$$

The kernel $V(\mathbf{p},\mathbf{q};M)$ in Eq. (11) is the quasipotential operator of the quark-antiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. An important role in this construction is played by the Lorentz structure of the confining quark-antiquark interaction in the meson. In constructing the quasipotential of the quark-antiquark interaction we have assumed that the effective interaction is the sum of the usual one-gluon exchange term and the mixture of vector and scalar linear confining potentials. The quasipotential is then defined by [25]

$$V(\mathbf{p},\mathbf{q};M)=\bar{u}_q(p)\bar{u}_Q(-p)\mathcal{V}(\mathbf{p},\mathbf{q};M)u_q(q)u_Q(-q),\tag{14}$$

with

$$\mathcal{V}(\mathbf{p},\mathbf{q};M)=\frac{4}{3}\alpha_sD_{\mu\nu}(\mathbf{k})\gamma_q^\mu\gamma_Q^\nu+V_{\mathrm{conf}}^V(\mathbf{k})\Gamma_q^\mu\Gamma_{Q;\mu}+V_{\mathrm{conf}}^S(\mathbf{k}),$$

where α_s is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge, and $\mathbf{k}=\mathbf{p}-\mathbf{q}$; γ_μ and $u(p)$ are the Dirac matrices and spinors

$$u^\lambda(p)=\sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}}\begin{pmatrix}1\\ \dfrac{\boldsymbol{\sigma}\mathbf{p}}{\epsilon(p)+m}\end{pmatrix}\chi^\lambda\tag{15}$$

with $\epsilon(p)=\sqrt{\mathbf{p}^2+m^2}$. The effective long-range vector vertex is given by

$$\Gamma_\mu(\mathbf{k})=\gamma_\mu+\frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu,\tag{16}$$

where κ is the Pauli interaction constant characterizing the nonperturbative anomalous chromomagnetic moment of quarks. Vector and scalar confining potentials in the nonrelativistic limit reduce to

$$V_{\mathrm{conf}}^V(r)=(1-\varepsilon)(Ar+B),\quad V_{\mathrm{conf}}^S(r)=\varepsilon(Ar+B),\tag{17}$$

reproducing

$$V_{\mathrm{conf}}(r)=V_{\mathrm{conf}}^S(r)+V_{\mathrm{conf}}^V(r)=Ar+B,\tag{18}$$

where ε is the mixing coefficient.

The quasipotential for the heavy quarkonia, expanded in v^2/c^2, can be found in Refs. [25,26] and for heavy-light mesons in Ref. [14]. All the parameters of our model, such as quark masses, parameters of the linear confining potential, mixing coefficient ε and anomalous chromomagnetic quark moment κ, were fixed from the analysis of heavy quarkonia masses [25] and radiative decays [27]. The quark masses $m_b=4.88$ GeV, $m_c=1.55$ GeV, $m_s=0.50$ GeV, $m_{u,d}=0.33$ GeV and the parameters of the linear potential $A=0.18$ GeV^2 and $B=-0.30$ GeV have the usual quark model values. In Ref. [17] we have considered the expansion of the matrix elements of weak heavy quark currents between pseudoscalar and vector meson ground states up to the

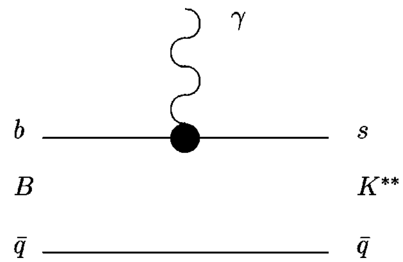

FIG. 1. Lowest order vertex function $\Gamma^{(1)}$ corresponding to Eq. (20).

second order in inverse powers of the heavy quark masses. It has been found that the general structure of the leading, first, and second order $1/m_Q$ corrections in our relativistic model is in accord with the predictions of HQET. The heavy quark symmetry and QCD impose rigid constraints on the parameters of the long-range potential in our model. The analysis of the first order corrections [17] fixes the value of the Pauli interaction constant $\kappa=-1$. The same value of κ was found previously from the fine splitting of heavy quarkonia ^3P_J-states [25]. The value of the parameter characterizing the mixing of vector and scalar confining potentials, $\varepsilon=-1$, was found from the analysis of the second order corrections [17]. This value is very close to the one determined from considering radiative decays of heavy quarkonia [27].

IV. RARE RADIATIVE $B\to K^{**}\gamma$ DECAY FORM FACTORS

In the quasipotential approach, the matrix element of the weak current $J_\mu=\bar{s}(i/2)k^\nu\sigma_{\mu\nu}(1+\gamma^5)b$ between the states of a B meson and an orbitally excited K^{**} meson has the form [29]

$$\langle K^{**}|J_\mu(0)|B\rangle=\int\frac{d^3p\,d^3q}{(2\pi)^6}\bar{\Psi}_{K^{**}}(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi_B(\mathbf{q}), \tag{19}$$

where $\Gamma_\mu(\mathbf{p},\mathbf{q})$ is the two-particle vertex function and $\Psi_{B,K^{**}}$ are the meson wave functions projected onto the positive energy states of quarks and boosted to the moving reference frame. The contributions to Γ come from Figs. 1 and 2. The contribution $\Gamma^{(2)}$ is the consequence of the projection onto the positive-energy states. Note that the form of the relativistic corrections resulting from the vertex function $\Gamma^{(2)}$ explicitly depends on the Lorentz structure of the $q\bar{q}$ interaction. The vertex functions are given by

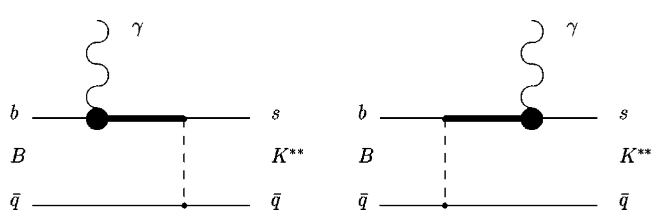

FIG. 2. Vertex function $\Gamma^{(2)}$ corresponding to Eq. (21). Dashed lines represent the interaction operator $\mathcal{V}$ in Eq. (14). Bold lines denote the negative-energy part of the quark propagator.

$$\Gamma_\mu^{(1)}(\mathbf{p},\mathbf{q})=\bar{u}_s(p_1)\frac{i}{2}\sigma_{\mu\nu}k^\nu(1+\gamma^5)u_b(q_1)(2\pi)^3\delta(\mathbf{p}_2-\mathbf{q}_2), \tag{20}$$

and

$$\begin{aligned}\Gamma_\mu^{(2)}(\mathbf{p},\mathbf{q})=\bar{u}_s(p_1)\bar{u}_q(p_2)\frac{1}{2}\Bigg\{&i\sigma_{1\mu\nu}k_\nu(1+\gamma_1^5)\\&\times\frac{\Lambda_b^{(-)}(k_1)}{\epsilon_b(k_1)+\epsilon_b(p_1)}\gamma_1^0\mathcal{V}(\mathbf{p}_2-\mathbf{q}_2)+\mathcal{V}(\mathbf{p}_2-\mathbf{q}_2)\\&\times\frac{\Lambda_s^{(-)}(k_1')}{\epsilon_s(k_1')+\epsilon_s(q_1)}\gamma_1^0 i\sigma_{1\mu\nu}k_\nu(1+\gamma_1^5)\Bigg\}\\&\times u_b(q_1)u_q(q_2),\end{aligned} \tag{21}$$

where $\mathbf{k}_1=\mathbf{p}_1-\mathbf{\Delta}$; $\mathbf{k}_1'=\mathbf{q}_1+\mathbf{\Delta}$; $\mathbf{\Delta}=\mathbf{p}_{K^{**}}-\mathbf{p}_B$;

$$\Lambda^{(-)}(p)=\frac{\epsilon(p)-(m\gamma^0+\gamma^0(\boldsymbol{\gamma}\mathbf{p}))}{2\epsilon(p)}.$$

The wave functions of P-wave K^{**} mesons at rest can be parametrized either through the wave functions of the states $^1P_1$, $^3P_{0,1,2}$ used in quark models for quarkonia (LS-coupling scheme) or through the wave functions $K(1/2)$ and $K(3/2)$ used in HQET (js-coupling scheme). The structure of the wave functions for the states with 0^+ and 2^+ quantum numbers is the same in both parametrizations, while two real states with 1^+ quantum numbers are different mixtures of states in these parametrizations. Experiment shows that $K_1(1400)$ and $K_1^*(1270)$ mesons are nearly equal mixtures of $^1P_1$ and $^3P_1$ quark model states [28]. As a result the HQET parametrization turns out to be more appropriate since the real $K_1(1400)$ and $K_1^*(1270)$ mesons almost coincide with the corresponding states in js-coupling scheme. The wave functions at rest in HQET parametrization are given by

$$\Psi_{K^{**}}(\mathbf{p})\equiv\Psi_{K(j)}^{JM}(\mathbf{p})=\mathcal{Y}_j^{JM}\psi_{K(j)}(\mathbf{p}), \tag{22}$$

where J and M are the meson total angular momentum and its projection, while j is the u,d-quark angular momentum; $\psi_{K(j)}(\mathbf{p})$ is the radial part of the wave function. The spin-angular momentum part $\mathcal{Y}_j^{JM}$ has the following form:

$$\begin{aligned}\mathcal{Y}_j^{JM}=\sum_{\sigma_Q\sigma_q}&\left\langle j\,M-\sigma_Q,\frac{1}{2}\sigma_Q\middle|J\,M\right\rangle\\&\times\left\langle 1\,M-\sigma_Q-\sigma_q,\frac{1}{2}\sigma_q\middle|j\,M-\sigma_Q\right\rangle\\&\times Y_1^{M-\sigma_Q-\sigma_q}\chi_Q(\sigma_Q)\chi_q(\sigma_q).\end{aligned} \tag{23}$$

Here $\langle j_1\,m_1,\,j_2\,m_2|J\,M\rangle$ are the Clebsch-Gordan coefficients, Y_l^m are the spherical harmonics, and $\chi(\sigma)$ (where $\sigma=\pm1/2$) are spin wave functions:

$$\chi(1/2)=\begin{pmatrix}1\\0\end{pmatrix},\qquad\chi(-1/2)=\begin{pmatrix}0\\1\end{pmatrix}.$$

Then

$$\Psi^M_{K_1(1400)} = \Psi^{1M}_{K(3/2)} \cos\phi + \Psi^{1M}_{K(1/2)} \sin\phi,$$
$$\Psi^M_{K_1^*(1270)} = \Psi^{1M}_{K(1/2)} \cos\phi - \Psi^{1M}_{K(3/2)} \sin\phi, \tag{24}$$

where ϕ is a small mixing angle. We have calculated the wave functions of orbitally excited K^{**} mesons in our model by the numerical solution of Eq. (11) with the quasipotential (14) expanded in inverse powers of quark energies. Such an expansion is more adequate for K mesons than the usual nonrelativistic expansion since the velocities of light u,d,s quarks are highly relativistic. The calculated spin-averaged P-wave K meson masses as well as spin-orbit splittings are consistent with experimental values. The obtained value of the mixing angle in Eq. (24) also agrees with the experiment and is approximately equal to $\phi \approx 4°$. In the following we will use the functions (22) for decay form factor calculations assuming that the physical form factors for $B \to K_1^{(*)}\gamma$ decays are related to the calculated ones (denoted by a tilde) by

$$t_+ = \tilde{t}_+ \cos\phi + \tilde{s}_+ \sin\phi,$$
$$s_+ = \tilde{s}_+ \cos\phi - \tilde{t}_+ \sin\phi. \tag{25}$$

It is important to note that the wave functions entering the weak current matrix element (19) cannot be both in the rest frame. In the B meson rest frame, the K^{**} meson is moving with the recoil momentum $\mathbf{\Delta}$. The wave function of the moving K^{**} meson $\Psi_{K^{**}\mathbf{\Delta}}$ is connected with the K^{**} wave function in the rest frame $\Psi_{K^{**}\mathbf{0}} \equiv \Psi_{K^{**}}$ by the transformation [29]

$$\Psi_{K^{**}\mathbf{\Delta}}(\mathbf{p}) = D_s^{1/2}(R^W_{L_{\mathbf{\Delta}}}) D_q^{1/2}(R^W_{L_{\mathbf{\Delta}}}) \Psi_{K^{**}\mathbf{0}}(\mathbf{p}), \tag{26}$$

where R^W is the Wigner rotation, $L_{\mathbf{\Delta}}$ is the Lorentz boost from the meson rest frame to a moving one, and the rotation matrix $D^{1/2}(R)$ in spinor representation is given by

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} D^{1/2}_{s,q}(R^W_{L_{\mathbf{\Delta}}}) = S^{-1}(\mathbf{p}_{s,q}) S(\mathbf{\Delta}) S(\mathbf{p}), \tag{27}$$

where

$$S(\mathbf{p}) = \sqrt{\frac{\epsilon(p)+m}{2m}} \left(1 + \frac{\boldsymbol{\alpha}\mathbf{p}}{\epsilon(p)+m}\right)$$

is the usual Lorentz transformation matrix of the four spinor.

We substitute the vertex functions $\Gamma^{(1)}$ and $\Gamma^{(2)}$ given by Eqs. (20) and (21) in the decay matrix element (19) and take into account the wave function transformation (26). The resulting structure of this matrix element is rather complicated, because it is necessary to integrate both over d^3p and d^3q. The δ function in expression (20) permits one to perform one of these integrations and thus this contribution can be easily calculated. The calculation of the vertex function $\Gamma^{(2)}$ contribution is more difficult. Here, instead of a δ function, we have a complicated structure, containing the $q\bar{q}$ interaction operator $\mathcal{V}$. However, we can expand this contribution in the inverse powers of the heavy b quark mass and large recoil momentum $|\mathbf{\Delta}| \sim m_b/2$ of the final K^{**} meson. Such an expansion is carried out up to the second order.[1] Then we use the quasipotential equation in order to perform one of the integrations in the current matrix element. As a result we get for the form factors the following expressions with $\kappa = -1$:

$$\tilde{t}_+(0) = \tilde{t}^{(1)}_+(0) + (1-\varepsilon)\tilde{t}^{(2)V}_+(0) + \varepsilon \tilde{t}^{(2)S}_+(0), \tag{28}$$

$$\begin{aligned}\tilde{t}^{(1)}_+(0) = {} & \frac{1}{3\sqrt{2}} \sqrt{\frac{E_{K(3/2)}}{M_B}} \frac{|\mathbf{\Delta}|}{E_{K(3/2)}+M_{K(3/2)}} \int \frac{d^3p}{(2\pi)^3} \bar{\psi}_{K(3/2)}\left(\mathbf{p} + \frac{2\epsilon_q}{E_{K(3/2)}+M_{K(3/2)}}\mathbf{\Delta}\right) \sqrt{\frac{\epsilon_s(p+\Delta)+m_s}{2\epsilon_s(p+\Delta)}} \sqrt{\frac{\epsilon_b(p)+m_b}{2\epsilon_b(p)}} \\ & \times \Bigg\{ -3(E_{K(3/2)}+M_{K(3/2)}) \frac{(\mathbf{p}\cdot\mathbf{\Delta})}{p\mathbf{\Delta}^2} \left[1 + \frac{M_B - E_{K(3/2)}}{\epsilon_s(p+\Delta)+m_s}\right] + \left[\frac{p}{\epsilon_q(p)+m_q} - \frac{p}{\epsilon_s(p+\Delta)+m_s}\right] \\ & \times \left[1 + \frac{M_B - E_{K(3/2)}}{\epsilon_s(p+\Delta)+m_s} + \frac{\mathbf{p}^2}{[\epsilon_s(p+\Delta)+m_s][\epsilon_b(p)+m_b]}\right] - 2\frac{M_B+M_{K(3/2)}}{M_B-M_{K(3/2)}} \frac{p}{\epsilon_s(p+\Delta)+m_s} \Bigg\} \psi_B(\mathbf{p}), \end{aligned} \tag{29}$$

[1]This means that in expressions for $\tilde{t},\tilde{s},g^{(2)V}_+(0)$ and $\tilde{t},\tilde{s},g^{(2)S}_+(0)$ we neglect terms proportional to the third order product of small binding energies and ratios $\mathbf{p}^2/\epsilon_s^3(\Delta)$, $\mathbf{p}^2/\epsilon_b^3(\Delta)$ as well as higher order terms.

$$\tilde{t}_+^{(2)V}(0)=\frac{1}{3\sqrt{2}}\sqrt{\frac{E_{K(3/2)}}{M_B}}\frac{|\mathbf{\Delta}|}{E_{K(3/2)}+M_{K(3/2)}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_{K(3/2)}\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K(3/2)}+M_{K(3/2)}}\mathbf{\Delta}\right)$$
$$\times\sqrt{\frac{\epsilon_s(\Delta)+m_s}{2\epsilon_s(\Delta)}}\Bigg\{\left[3(E_{K(3/2)}+M_{K(3/2)})\frac{(\mathbf{p}\cdot\mathbf{\Delta})}{p\mathbf{\Delta}^2}-\frac{p}{\epsilon_q(p)+m_q}\right]\frac{\epsilon_s(\Delta)-m_s}{2\epsilon_s(\Delta)[\epsilon_s(\Delta)+m_s]}$$
$$\times\Bigg(M_B-\epsilon_b(p)-\epsilon_q(p)+\frac{M_B-E_{K(3/2)}}{\epsilon_s(\Delta)+m_s}\Bigg[M_{K(3/2)}-\epsilon_s\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K(3/2)}+M_{K(3/2)}}\mathbf{\Delta}\right)$$
$$-\epsilon_s\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K(3/2)}+M_{K(3/2)}}\mathbf{\Delta}\right)\Bigg]\Bigg)+\frac{p}{\epsilon_q(p)+m_q}\Bigg[\frac{M_B-\epsilon_b(p)-\epsilon_q(p)}{\epsilon_b(\Delta)+m_b}+3\frac{\epsilon_s(\Delta)-m_s}{2\epsilon_s(\Delta)[\epsilon_s(\Delta)+m_s]}$$
$$\times\left(M_{K(3/2)}-\epsilon_s\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K(3/2)}+M_{K(3/2)}}\mathbf{\Delta}\right)-\epsilon_s\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K(3/2)}+M_{K(3/2)}}\mathbf{\Delta}\right)\right)\Bigg]\Bigg\}\psi_B(\mathbf{p}), \tag{30}$$

$$\tilde{t}_+^{(2)S}(0)=\frac{1}{3\sqrt{2}}\sqrt{\frac{E_{K(3/2)}}{M_B}}\frac{|\mathbf{\Delta}|}{E_{K(3/2)}+M_{K(3/2)}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_{K(3/2)}\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K(3/2)}+M_{K(3/2)}}\mathbf{\Delta}\right)$$
$$\times\sqrt{\frac{\epsilon_s(\Delta)+m_s}{2\epsilon_s(\Delta)}}\Bigg\{\left[-3(E_{K(3/2)}+M_{K(3/2)})\frac{(\mathbf{p}\cdot\mathbf{\Delta})}{p\mathbf{\Delta}^2}+\frac{p}{\epsilon_q(p)+m_q}\right]\frac{\epsilon_s(\Delta)-m_s}{2\epsilon_s(\Delta)[\epsilon_s(\Delta)+m_s]}$$
$$\times\Bigg[M_B-\epsilon_b(p)-\epsilon_q(p)-\frac{M_B-E_{K(3/2)}}{\epsilon_s(\Delta)+m_s}\Bigg(M_{K(3/2)}-\epsilon_s\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K(3/2)}+M_{K(3/2)}}\mathbf{\Delta}\right)$$
$$-\epsilon_s\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K(3/2)}+M_{K(3/2)}}\mathbf{\Delta}\right)\Bigg)\Bigg]\Bigg\}\psi_B(\mathbf{p}), \tag{31}$$

$$\tilde{s}_+(0)=\tilde{s}_+^{(1)}(0)+(1-\varepsilon)\tilde{s}_+^{(2)V}(0)+\varepsilon\tilde{s}_+^{(2)S}(0), \tag{32}$$

$$\tilde{s}_+^{(1)}(0)=\frac{1}{3}\sqrt{\frac{E_{K(1/2)}}{M_B}}\frac{|\mathbf{\Delta}|}{E_{K(1/2)}+M_{K(1/2)}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_{K(1/2)}\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K(1/2)}+M_{K(1/2)}}\mathbf{\Delta}\right)$$
$$\times\sqrt{\frac{\epsilon_s(p+\Delta)+m_s}{2\epsilon_s(p+\Delta)}}\sqrt{\frac{\epsilon_b(p)+m_b}{2\epsilon_b(p)}}\Bigg\{-3(E_{K(1/2)}+M_{K(1/2)})\frac{(\mathbf{p}\cdot\mathbf{\Delta})}{p\mathbf{\Delta}^2}\left[1+\frac{M_B-E_{K(1/2)}}{\epsilon_s(p+\Delta)+m_s}\right]$$
$$-\left[\frac{2p}{\epsilon_q(p)+m_q}-\frac{2p}{\epsilon_s(p+\Delta)+m_s}\right]\left[1+\frac{M_B-E_{K(1/2)}}{\epsilon_s(p+\Delta)+m_s}+\frac{\mathbf{p}^2}{[\epsilon_s(p+\Delta)+m_s][\epsilon_b(p)+m_b]}\right]$$
$$-\frac{M_B+M_{K(1/2)}}{M_B-M_{K(1/2)}}\left[\frac{p}{\epsilon_b(p)+m_b}\left(1+\frac{M_B-E_{K(1/2)}}{\epsilon_s(p+\Delta)+m_s}\right)+\frac{3p}{\epsilon_s(p+\Delta)+m_s}\right]\Bigg\}\psi_B(\mathbf{p}), \tag{33}$$

$$\tilde{s}_+^{(2)V}(0)=\frac{1}{3}\sqrt{\frac{E_{K(1/2)}}{M_B}}\frac{|\mathbf{\Delta}|}{E_{K(1/2)}+M_{K(1/2)}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_{K(1/2)}\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K(1/2)}+M_{K(1/2)}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_s(\Delta)+m_s}{2\epsilon_s(\Delta)}}$$
$$\times\left\{\left[3(E_{K(1/2)}+M_{K(1/2)})\frac{(\mathbf{p}\cdot\mathbf{\Delta})}{p\mathbf{\Delta}^2}+\frac{2p}{\epsilon_q(p)+m_q}\right]\frac{\epsilon_s(\Delta)-m_s}{2\epsilon_s(\Delta)[\epsilon_s(\Delta)+m_s]}\right.$$
$$\times\left(M_B-\epsilon_b(p)-\epsilon_q(p)+\frac{M_B-E_{K(1/2)}}{\epsilon_s(\Delta)+m_s}\right.$$
$$\times\left.\left[M_{K(1/2)}-\epsilon_s\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K(1/2)}+M_{K(1/2)}}\mathbf{\Delta}\right)-\epsilon_s\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K(1/2)}+M_{K(1/2)}}\mathbf{\Delta}\right)\right]\right)-\frac{p}{2(\epsilon_q(p)+m_q)}\left[\frac{1}{\epsilon_b(\Delta)+m_b}\right.$$
$$\times\left([M_B-\epsilon_b(p)-\epsilon_q(p)]\left(3\frac{M_B+M_{K(1/2)}}{M_B-M_{K(1/2)}}-1\right)+\frac{M_B+M_{K(1/2)}}{M_B-M_{K(1/2)}}\left[M_{K(1/2)}-\epsilon_s\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K(1/2)}+M_{K(1/2)}}\mathbf{\Delta}\right)\right.\right.$$
$$\left.\left.-\epsilon_s\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K(1/2)}+M_{K(1/2)}}\mathbf{\Delta}\right)\right]\right)+\frac{3}{2\epsilon_s(\Delta)}\left(\frac{M_B+M_{K(1/2)}}{M_B-M_{K(1/2)}}[M_B-\epsilon_b(p)-\epsilon_q(p)]+\left(\frac{M_B+M_{K(1/2)}}{M_B-M_{K(1/2)}}\right.\right.$$
$$\left.\left.\left.\left.-2\frac{\epsilon_s(\Delta)-m_s}{\epsilon_s(\Delta)+m_s}\right)\left[M_{K(1/2)}-\epsilon_s\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K(1/2)}+M_{K(1/2)}}\mathbf{\Delta}\right)-\epsilon_s\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K(1/2)}+M_{K(1/2)}}\mathbf{\Delta}\right)\right]\right)\right]\right\}\psi_B(\mathbf{p}), \qquad (34)$$

$$\tilde{s}_+^{(2)S}(0)=\frac{1}{3}\sqrt{\frac{E_{K(1/2)}}{M_B}}\frac{|\mathbf{\Delta}|}{E_{K(1/2)}+M_{K(1/2)}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_{K(1/2)}\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K(1/2)}+M_{K(1/2)}}\mathbf{\Delta}\right)$$
$$\times\sqrt{\frac{\epsilon_s(\Delta)+m_s}{2\epsilon_s(\Delta)}}\left\{\left[-3[E_{K(1/2)}+M_{K(1/2)}]\frac{(\mathbf{p}\cdot\mathbf{\Delta})}{p\mathbf{\Delta}^2}-\frac{2p}{\epsilon_q(p)+m_q}\right]\frac{\epsilon_s(\Delta)-m_s}{2\epsilon_s(\Delta)[\epsilon_s(\Delta)+m_s]}\right.$$
$$\times\left[M_B-\epsilon_b(p)-\epsilon_q(p)-\frac{M_B-E_{K(1/2)}}{\epsilon_s(\Delta)+m_s}\left(M_{K(1/2)}-\epsilon_s\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K(1/2)}+M_{K(1/2)}}\mathbf{\Delta}\right)\right.\right.$$
$$\left.\left.\left.-\epsilon_s\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K(1/2)}+M_{K(1/2)}}\mathbf{\Delta}\right)\right)\right]\right\}\psi_B(\mathbf{p}), \qquad (35)$$

$$g_+(0)=g_+^{(1)}(0)+(1-\varepsilon)g_+^{(2)V}(0)+\varepsilon g_+^{(2)S}(0), \qquad (36)$$

$$g_+^{(1)}(0)=\frac{1}{\sqrt{3}}\sqrt{\frac{E_{K_2^*}}{M_B}}\frac{M_{K_2^*}}{E_{K_2^*}+M_{K_2^*}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_{K_2^*}\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K_2^*}+M_{K_2^*}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_s(p+\Delta)+m_s}{2\epsilon_s(p+\Delta)}}\sqrt{\frac{\epsilon_b(p)+m_b}{2\epsilon_b(p)}}$$
$$\times\left\{-3(E_{K_2^*}+M_{K_2^*})\frac{(\mathbf{p}\cdot\mathbf{\Delta})}{p\mathbf{\Delta}^2}\left[1+\frac{M_B-E_{K_2^*}}{\epsilon_s(p+\Delta)+m_s}\right]+\left[\frac{p}{\epsilon_q(p)+m_q}-\frac{p}{\epsilon_s(p+\Delta)+m_s}\right]\right.$$
$$\left.\times\left[1+\frac{M_B-E_{K_2^*}}{\epsilon_s(p+\Delta)+m_s}-\frac{\mathbf{p}^2}{[\epsilon_s(p+\Delta)+m_s][\epsilon_b(p)+m_b]}\right]\right\}\psi_B(\mathbf{p}), \qquad (37)$$

$$g_+^{(2)V}(0)=\frac{1}{\sqrt{3}}\sqrt{\frac{E_{K_2^*}}{M_B}}\frac{M_{K_2^*}}{E_{K_2^*}+M_{K_2^*}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_{K_2^*}\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K_2^*}+M_{K_2^*}}\mathbf{\Delta}\right)$$
$$\times\sqrt{\frac{\epsilon_s(\Delta)+m_s}{2\epsilon_s(\Delta)}}\Bigg\{3(E_{K_2^*}+M_{K_2^*})\frac{(\mathbf{p}\cdot\mathbf{\Delta})}{p\mathbf{\Delta}^2}\,\frac{M_B-\epsilon_b(p)-\epsilon_q(p)}{2[\epsilon_s(\Delta)+m_s]}$$
$$-\frac{p}{\epsilon_q(p)+m_q}\,\frac{1}{2[\epsilon_s(\Delta)+m_s]^2}\Bigg((M_B+M_{K_2^*})[M_B-\epsilon_b(p)-\epsilon_q(p)]+(E_{K_2^*}+M_{K_2^*})$$
$$\times\left[M_{K_2^*}-\epsilon_s\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K_2^*}+M_{K_2^*}}\mathbf{\Delta}\right)-\epsilon_s\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K_2^*}+M_{K_2^*}}\mathbf{\Delta}\right)\right]\Bigg)\Bigg\}\psi_B(\mathbf{p}), \tag{38}$$

$$g_+^{(2)S}(0)=\frac{1}{\sqrt{3}}\sqrt{\frac{E_{K_2^*}}{M_B}}\frac{M_{K_2^*}}{E_{K_2^*}+M_{K_2^*}}\int\frac{d^3p}{(2\pi)^3}\bar{\psi}_{K_2^*}\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K_2^*}+M_{K_2^*}}\mathbf{\Delta}\right)\sqrt{\frac{\epsilon_s(\Delta)+m_s}{2\epsilon_s(\Delta)}}$$
$$\times\Bigg\{\left[-3(E_{K_2^*}+M_{K_2^*})\frac{(\mathbf{p}\cdot\mathbf{\Delta})}{p\mathbf{\Delta}^2}+\frac{p}{\epsilon_q(p)+m_q}\right]\Bigg(\frac{\epsilon_s(\Delta)-m_s}{2\epsilon_s(\Delta)[\epsilon_s(\Delta)+m_s]}\Bigg[M_{K_2^*}-\epsilon_s\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K_2^*}+M_{K_2^*}}\mathbf{\Delta}\right)$$
$$-\epsilon_s\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K_2^*}+M_{K_2^*}}\mathbf{\Delta}\right)\Bigg]+\frac{M_B-E_{K_2^*}}{2[\epsilon_s(\Delta)+m_s]^2}\Bigg[M_B+M_{K_2^*}-\epsilon_b(p)-\epsilon_q(p)-\epsilon_s\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K_2^*}+M_{K_2^*}}\mathbf{\Delta}\right)$$
$$-\epsilon_s\left(\mathbf{p}+\frac{2\epsilon_q}{E_{K_2^*}+M_{K_2^*}}\mathbf{\Delta}\right)\Bigg]\Bigg)\Bigg\}\psi_B(\mathbf{p}), \tag{39}$$

where the superscripts "(1)" and "(2)" correspond to contributions coming from Figs. 1 and 2, S and V mean the scalar and vector potentials in Eq. (17), $\psi_{K^{**},B}$ are radial parts of the wave functions. Since $M_{K(1/2)}$ and $M_{K(3/2)}$ almost coincide with the physical axial-vector meson masses $M_{K_1^{(*)}}$ and M_{K_1} we use the latter for numerical calculations. The recoil momentum and the energy of the K^{**} meson are given by

$$|\mathbf{\Delta}|=\frac{M_B^2-M_{K^{**}}^2}{2M_B};\quad E_{K^{**}}=\frac{M_B^2+M_{K^{**}}^2}{2M_B}. \tag{40}$$

V. RESULTS AND DISCUSSION

We can check the consistency of our resulting formulas by taking the formal limit of b and s quark masses going to infinity.[2] In this limit according to HQET [30] the functions

$$\xi_F(w)=\frac{2\sqrt{M_BM_{K_2^*}}}{M_B+M_{K_2^*}}g_+(w)=\frac{2\sqrt{M_BM_{K_1}}}{M_B-M_{K_1}}\frac{\sqrt{6}}{w+1}\tilde{t}_+(w) \tag{41}$$

and

$$\xi_E(w)=\frac{2\sqrt{M_BM_{K_1^*}}}{M_B-M_{K_1^*}}\tilde{s}_+(w),\quad w=\frac{M_B^2+M_{K^{**}}^2-k^2}{2M_BM_{K^{**}}}, \tag{42}$$

should coincide with the Isgur-Wise functions $\tau(w)$ and $\zeta(w)$ for semileptonic B decays to orbitally excited D mesons, $B\to D^{**}e\nu$. Such semileptonic decays have been considered by us in Ref. [31]. Taking the formal limit $m_b\to\infty$, $m_s\to\infty$ in Eqs. (28)–(39) and using definitions (41), (42) we find

[2]As was noted above, such a limit is justified only for the b quark.

TABLE I. Theoretical predictions and experimental data for the branching fractions ($\times 10^{-5}$) and their ratios $R_{K^*} \equiv \mathrm{Br}(B \to K^* \gamma)/\mathrm{Br}(B \to X_s \gamma)$, $R_{K_i^{(*)}} \equiv \mathrm{Br}(B \to K_i^{(*)} \gamma)/\mathrm{Br}(B \to X_s \gamma)$ $(i=1,2)$, $r \equiv \mathrm{Br}(B \to K_2^* \gamma)/\mathrm{Br}(B \to K^* \gamma)$. Our values for the $B \to K^* \gamma$ decay are taken from Ref. [10].

Value	Ours	Ref. [5]	Ref. [6]	Ref. [7]	Ref. [8]	Exp. CLEO [2]	Exp. Belle [3]
$\mathrm{Br}(B \to K^*(892)\gamma)$	4.5 ± 1.5	1.35	$1.4-4.9$	$0.5-0.8$	4.71 ± 1.79	$4.55^{+0.72}_{-0.68} \pm 0.34$ [a]	$4.96 \pm 0.67 \pm 0.45$ [a]
						$3.76^{+0.89}_{-0.83} \pm 0.28$ [b]	$3.89 \pm 0.93 \pm 0.41$ [b]
R_{K^*} (%)	15 ± 3	4.5	$3.5-12.2$	$1.6-2.5$	16.8 ± 6.4		
$B \to K_0^*(1430)\gamma$				forbidden			
$\mathrm{Br}(B \to K_1^*(1270)\gamma)$	0.45 ± 0.15	1.1	$1.8-4.0$	$0.3-1.4$	1.20 ± 0.44		
$R_{K_1^*}$ (%)	1.5 ± 0.5	3.8	$4.5-10.1$	$0.9-4.5$	4.3 ± 1.6		
$\mathrm{Br}(B \to K_1(1400)\gamma)$	0.78 ± 0.18	0.7	$2.4-5.2$	$0.1-0.6$	0.58 ± 0.26		
R_{K_1} (%)	2.6 ± 0.6	2.2	$6.0-13.0$	$0.4-2.0$	2.1 ± 0.9		
$\mathrm{Br}(B \to K_2^*(1430)\gamma)$	1.7 ± 0.6	1.8	$6.9-14.8$	$0.4-1.0$	1.73 ± 0.80	$1.66^{+0.59}_{-0.53} \pm 0.13$	$1.89 \pm 0.56 \pm 0.18$
$R_{K_2^*}$ (%)	5.7 ± 1.2	6.0	$17.3-37.1$	$1.3-3.2$	6.2 ± 2.9		
r	0.38 ± 0.08	1.3	$3.0-4.9$	$0.8-1.3$	0.37 ± 0.10	$0.39^{+0.15}_{-0.13}$	

[a] $B^0 \to K^{*0}\gamma$.
[b] $B^+ \to K^{*+}\gamma$.

$$\xi_F(w) = \sqrt{\frac{2}{3}} \frac{1}{(w+1)^{3/2}} \int \frac{d^3p}{(2\pi)^3} \bar{\psi}_{K(3/2)}\left(\mathbf{p} + \frac{2\epsilon_q}{M_{K(3/2)}(w+1)}\mathbf{\Delta}\right)\left[-3M_{K(3/2)}(w+1)\frac{(\mathbf{p}\cdot\mathbf{\Delta})}{p\mathbf{\Delta}^2} + \frac{p}{\epsilon_q(p)+m_q}\right]\psi_B(\mathbf{p}), \tag{43}$$

$$\xi_E(w) = \frac{\sqrt{2}}{3} \frac{1}{(w+1)^{1/2}} \int \frac{d^3p}{(2\pi)^3} \bar{\psi}_{K(1/2)}\left(\mathbf{p} + \frac{2\epsilon_q}{M_{K(1/2)}(w+1)}\mathbf{\Delta}\right)\left[-3M_{K(1/2)}(w+1)\frac{(\mathbf{p}\cdot\mathbf{\Delta})}{p\mathbf{\Delta}^2} - 2\frac{p}{\epsilon_q(p)+m_q}\right]\psi_B(\mathbf{p}). \tag{44}$$

It is easy to verify that the equalities $\xi_F = \tau$ and $\xi_E = \zeta$ are implemented in our model if we also use the expansion in $(w-1)/(w+1)$ (w is a scalar product of four-velocities of the initial and final mesons), which is small for the $B \to D^{**} e \nu$ decay [31]. It is important to note that last terms in the square brackets of the expressions for the functions $\xi_F(w)$ (43) and $\xi_E(w)$ (44) result from the wave function transformation (26) associated with the relativistic rotation of the light quark spin (Wigner rotation) in passing to the moving reference frame. These terms are numerically important and lead to the suppression of the ξ_E form factor compared to ξ_F. Note that if we applied a simplified nonrelativistic quark model [8,30] these important contributions would be missing. Neglecting further the small difference between the wave functions $\psi_{K(1/2)}$ and $\psi_{K(3/2)}$, the following relation between ξ_F and ξ_E would be obtained [30]

$$\xi_E(w) = \frac{w+1}{\sqrt{3}} \xi_F(w). \tag{45}$$

However, we see that this relation is violated if the relativistic transformation properties of the wave function are taken into account.

The relations between the form factors of heavy-to-light semileptonic and rare radiative B decays emerging in the large recoil limit [11,12] are satisfied in our model [10,15]. Using Eq. (36) to calculate the ratio of the form factor $g_+(0)$ in the infinitely heavy b and s quark limit to the same form factor in the leading order of expansions in inverse powers of the heavy b quark mass and large recoil momentum $|\mathbf{\Delta}|$ we find that it is equal to $M_B/\sqrt{M_B^2 + M_{K_2^*}^2} \approx 0.965$. The corresponding ratio of form factors of the exclusive rare radiative B decay to the vector K^* meson $F_1(0)$ [see Eq. (23) of Ref. [10]] is equal to $M_B/\sqrt{M_B^2 + M_{K^*}^2} \approx 0.986$. Therefore we conclude that the form factor ratios $g_+(0)/F_1(0)$ in the leading order of these expansions differ by the factor $\sqrt{M_B^2 + M_{K^*}^2}/\sqrt{M_B^2 + M_{K_2^*}^2} \approx 0.98$. This is the consequence of the relativistic dynamics leading to the effective expansion in inverse powers of the s quark energy $\epsilon_s(p+\Delta) = \sqrt{(\mathbf{p}+\mathbf{\Delta})^2 + m_s^2}$, which is high in one case due to the large s quark mass and in the other one due to the large recoil momentum $\mathbf{\Delta}$. As a result both expansions give similar final expressions in the leading order. Thus we can expect that the ratio r of the B branching fractions to the tensor K_2^* and vector K^* mesons in our calculations should be close to the one found in the infinitely heavy s quark limit [8].

The results of numerical calculations using formulas (5)–(9), (25), (28)–(39) for $\varepsilon=-1$ are given in Table I. There we also show our previous predictions for the $B\to K^*\gamma$ decay [10]. Our results are confronted with other theoretical calculations [5–8] and recent experimental data [2]. The QCD sum rules predict (with 20% uncertainty) [21] Br($B\to K^*\gamma$)$=4.4\times10^{-5}\times(1+8\%)$, where the second term in the brackets is the estimate of the $1/m_c^2$ terms contribution. We find a good agreement of our predictions for decay rates with the experiment and estimates of Ref. [8] for the measured decay rates $B\to K^*\gamma$ and $B\to K_2^*\gamma$. Other theoretical calculations substantially disagree with data either for $B\to K^*\gamma$ [5,7] or for $B\to K_2^*\gamma$ [6] decay rates. Let us note that one of the main reasons of the too small values for $B\to K^*\gamma$ decay rates in quark models [5,7] is the use of the nonrelativistic expression for the momentum of the final meson in the argument of the wave function overlap [10]. As a result our predictions and those of Ref. [8] for the ratio r are well consistent with experiment, while the r estimates of Refs. [5,7] and [6] are several times larger than the experimental value (see Table I). As it was argued above, it is not accidental that r values in our and Ref. [8] approaches are close. The agreement of both predictions for branching fractions could be explained by some specific cancellation of finite s quark mass effects and relativistic corrections which were neglected in Ref. [8]. Though our numerical results for the measured decay rates agree with Ref. [8], we believe that our analysis is more consistent and reliable. We do not use the ill-defined limit $m_s\to\infty$, and our quark model consistently takes into account main relativistic effects, for example, the Lorentz transformation of the wave function of the final K^{**} meson [see Eq. (26)]. Such a transformation turns out to be very important and leads to the substantial reduction of $B\to K_1^*(1270)\gamma$ decay rate in our model. We see from Table I that our model predicts for the ratio Br($B\to K_1^*(1270)\gamma$)/Br($B\to K_1(1430)\gamma$) the value 0.7 ± 0.3 while Ref. [8] gives for this ratio a considerably larger value ~2, which is the consequence of the nonrelativistic quark model relation (45) between form factors ξ_F and ξ_E. Thus experimental measurement of Br($B\to K_1^*(1270)\gamma$) can discriminate between these predictions.

VI. CONCLUSIONS

In this paper we have investigated rare radiative B decays to orbitally excited K^{**} mesons in the framework of the relativistic quark model. The large value of the recoil momentum $|\mathbf{\Delta}|\sim m_b/2$ makes relativistic effects play a significant role and strongly increases the energy of the final meson. This effect considerably simplifies the analysis since it allows us to make an expansion both in inverse powers of the large b quark mass and in the large recoil momentum of the light final meson. Such an expansion has more firm theoretical grounds than the previously used expansion in inverse powers of the s quark mass [8,6], which is not heavy enough. We carried out this expansion up to the second order and calculated resulting form factors in our relativistic quark model. Rare radiative B decays to axial-vector $K_1^{(*)}$ and tensor K_2^* mesons have been considered. It was found that relativistic effects substantially influence decay form factors. Thus, the Wigner rotation of the light quark spin gives an important contribution, which leads to the suppression of the $B\to K_1^*(1270)\gamma$ decay rate. In the nonrelativistic quark model, where these effects are missing, the ratio of branching fractions Br($B\to K_1^*(1270)\gamma$)/Br($B\to K_1(1400)\gamma$) is equal to 2, while in our model it is substantially smaller and equal to 0.7 ± 0.3. It will be very interesting to test this conclusion experimentally. Our predictions for the branching fractions $B\to K^*\gamma$ and $B\to K_2^*\gamma$ as well as their ratio are in a good agreement with recent CLEO [2] and Belle [3] data.

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